Dynamics of Polish government yield curve

Testing Pure Expectations Hypothesis

26th April 2019

Advanced Econometrics Homework Presentations, Warsaw

Motivation and context

This task as a part of a bigger research undertaking

Context

Ongoing research on: *Morphology, Dynamics and Forecast Power of Polish Government Yield Curve*:

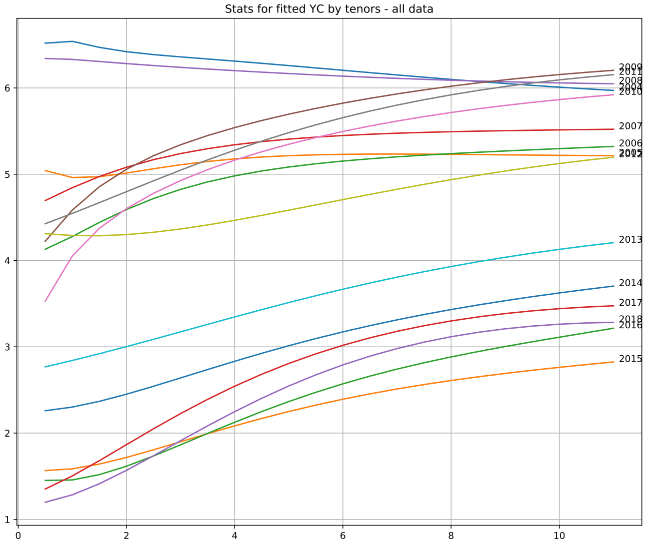
1. implement parsimonious (NSS) yield curve fitting techniques
2. analyse the dynamics of the yield curve via PCA, HPR decomposition, volume traded analysis
3. use synthetic fitted curves to infer on forecasting power of Polish yield curve using Fama-Bliss and Diebold-Lu frameworks

The particular task as homework 2: Testing Pure Expectations Hypothesis (PEH)

One of the renowned versions of PEH reads: *Forward rate is an unbiased predictor of the future spot rate*. Cf.: Bams&Wolff (2003), Bauer&Hamilton (2017)

Yield curves

Current zero-coupon bond yields may contain information on expected yields in the future



Extraction of market expectations

Implied forward rate is a main tool in interest rate instruments' valuations

Generic formula for forward yields (from zero-coupon yields)

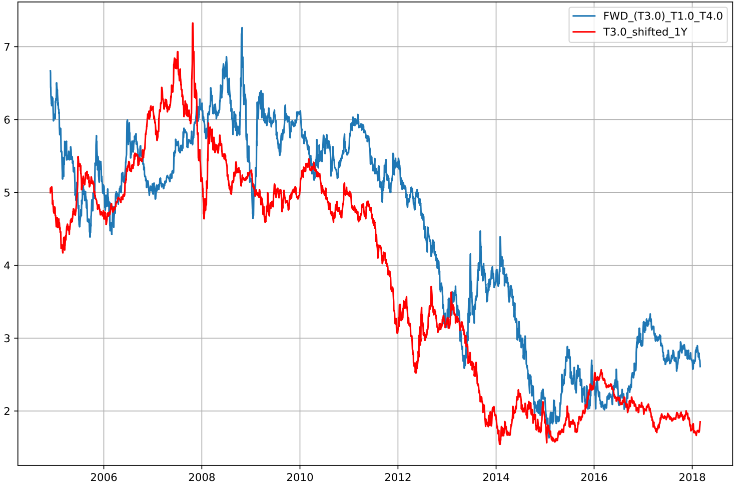
The $l - s$-year forward rate ($l > s$), starting s-years from t and ending l-years from t, prevailing in the market at time t is given by:

$$F_t(s,l) = \left(\frac{(1 + Y_y(l))^l}{(1 + Y_t(s))^s} \right)^{\frac{1}{l-s}} - 1$$

where: $Y_t(x)$ is a zero-coupon bond's yield which has x-years to maturity at time t.

How expectations align with reality?

Example of future and forward rates in 3-year tenor after 1 Y



Data

Derived in the previous parts of my research project

Dependant variable(s)

Future yield of τ-maturity observed in 1Y horizon from the moment of calculation of the forward rates (hence at: $t + 1Y$)

Regressor(s)

Forward yield of τ-maturity starting in 1Y time and calculated from zero-coupon yields observed at t

Long panel format

time dimension - 3228 consecutive fixing dates spanning from 2004 to 2019, **maturity dimension** - ten maturities ranging from $\tau = 1$ to $\tau = 10$

Generic regression specification

$$FutYield \sim CurrFwdYield$$

Pooled OLS

First findings on how expectations align with reality or vice-versa

What does it mean that PEH holds?

$$FutYield_{t,\tau} = \beta_0 + \beta_1 CurrFwdYield_{t,\tau} + \epsilon_{t,\tau}$$

β_0 should be statistically non-distinguishable from zero AND β_1 should be statistically non-distinguishable from one.

Results

```
Coefficients:
              Estimate Std. Error t-value Pr(>|t|)
(Intercept)  -0.144088   0.096812 -1.4883   0.1367
CurrFwdYield  0.923290   0.010805 85.4531   <2e-16 ***
```

S.E. calculated using robust covariance matrix estimators a la White for panel models (R: vcovHC())

Inference

We should reject the hypothesis of pure expectations as our β_1 is statistically significantly non-one.

Univariate regressions for each maturity

10 separate models

Results

	Dependent variable: FutYield									
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
CurrFwdYield	0.895***	0.942***	0.947***	0.934***	0.912***	0.891***	0.873***	0.860***	0.851***	0.847***
	(0.009)	(0.009)	(0.009)	(0.010)	(0.010)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)
Constant	-0.293***	-0.426***	-0.385***	-0.257***	-0.096*	0.063	0.200***	0.307***	0.382***	0.430***
	(0.039)	(0.040)	(0.043)	(0.047)	(0.050)	(0.053)	(0.055)	(0.056)	(0.058)	(0.059)

Inference

All tenor univariate regressions call for rejection of PEH, with all β_1 coefficients being siginificantly different from 1 and all β_0-s different from zero, but for $\tau = \{5, 6\}$.

Fixed effects

Maturity fixed effects

What does it mean that PEH holds?

$$FutYield_{t,\tau} = \beta_{0,\tau} + \beta_1 CurrFwdYield_{t,\tau} + \epsilon_{t,\tau}$$

Every $\beta_{0,\tau}$ should be statistically non-distinguishable from zero AND β_1 should be statistically non-distinguishable from one.

Results

Coefficients:

	Estimate	Std. Error	t-value	Pr(>|t|)	
CurrFwdYield	0.899756	0.011061	81.343	< 2.2e-16	***

	Estimate	Std. Error	t-value	Pr(>|t|)	
1	-0.311493	0.047183	-6.6018	4.124e-11	***
2	-0.245288	0.049140	-4.9916	6.018e-07	***
3	-0.174280	0.050814	-3.4297	0.0006049	***
4	-0.103034	0.052209	-1.9735	0.0484472	*
5	-0.036814	0.053350	-0.6900	0.4901680	
6	0.021528	0.054275	0.3966	0.6916336	
7	0.071027	0.055023	1.2909	0.1967579	
8	0.111830	0.055628	2.0103	0.0444046	*
9	0.144652	0.056121	2.5775	0.0099567	**
10	0.170453	0.056531	3.0152	0.0025701	**

Inference

Estimate of $\beta_1 = 0.8997$ is slightly lower than in pooled OLS (0.923374). Estimates for $\beta_{0,\tau}$ suggest strong dependence on τ: intercepts increase monotonically from -0.3115 for 1Y maturity to 0.1704 for 10Y bonds, which may indicate a maturity risk premia. S.E. are smaller than in univariate regressions. Honda Test for balanced panels: normal $= 264.6$, p-value $<$ 2.2e-16, hence the effects are significant.

Random effects

Maturity and time

Results

```
Coefficients:
              Estimate Std. Error z-value  Pr(>|z|)
(Intercept)  -0.083505   0.011061 -7.5494 4.374e-14 ***
CurrFwdYield  0.910203   0.011061 82.2879 < 2.2e-16 ***
```

```
        Hausman Test

data:  model_spec
chisq = 8404.8, df = 1, p-value < 2.2e-16
alternative hypothesis: one model is inconsistent
```

```
        Wooldridge's test for unobserved individual effects

data:  formula
z = 2.3684, p-value = 0.01787
alternative hypothesis: unobserved effect
```

Inference

Estimate of $\beta_1 = 0.9102$ is slightly higher than in fixed effects (0.89984). Estimate for $\beta_{0,\tau}$ is significantly different from zero. **Hausman** test calls for rejection of null hypothesis (when compared to fixed effects model), hence fixed effects are significant and random effects model is inconsistent. **Wooldridge**'s test also calls for rejection of the null hypothesis that *pooled OLS* is a good model. Rejection may follow from serial correlation of different kinds, and in particular, quoting Wooldridge (2002), *should not be interpreted as implying that the random effects error structure must be true.*

Yet another angle: Fama-Bliss regression

Do current forward yields forecasts interest rates?

What is Fama-Bliss type 1 regression?

$$FutYieldSpr_{t,\tau} = \beta_0 + \beta_1 CurrFwdYieldSpr_{t,\tau} + \epsilon_{t,\tau}$$

from: $$FutYield_{t,\tau} - Yield_{t,\tau} = \beta_0 + \beta_1 \left(CurrFwdYield_{t,\tau} - Yield_{t,\tau} \right) + \epsilon_{t,\tau}$$

should the yield curve has a forecasting power we would have β_1 should be statistically distinguishable from zero. As previously, all future or forward rates under our investigation start in 1Y time.

Results of pooled OLS

```
Coefficients:
                 Estimate Std. Error t-value Pr(>|t|)
(Intercept)     -0.282798   0.098637 -2.8671 0.004146 **
CurrFwdYieldSpr  0.239362   0.282718  0.8466 0.397198
```

S.E. calculated using robust covariance matrix estimators a la White for panel models (R: vcovHC())

Inference

No forecasting power in current forward rates (when the whole spectrum of maturities is concerned).

Conclusion and further research

Pure expectations hypothesis may be rejected … but what are the alternatives?

We reject PEH, but it is not clear what causes this rejection: existence of risk premia or unexpected excess yield (we have only a mixture of these two contained in β_0 coefficient estimators).

1. **Spanning hypothesis**. Yield curve spans all information relevant for forecasting future yields and returns and no variables other than the current curve are needed.
2. **Expectations hypothesis with constant risk premium**.
3. **Expectations hypothesis with liquidity premium a la Hicks**.

To test the above we need:

1. more sophisticated tools to separate unobservable effects of risk premia and unexpected excess yield via covariance structure (cf. Bams & Wolff (2003))
2. to redo the testing for 2Y-5Y starting yields
3. to prepare data for horizon realised returns